

MATH 1342 – STATISTICS
SECTION 7.3: USING THE CENTRAL LIMIT THEOREM (CLT)

- If you are being asked to find the probability of the mean, use the CLT for the mean.
- If you are being asked to find the probability of a sum or total, use the CLT for sums.
- This also applies to percentiles for means and sums.
- If you are being asked to find the probability of an individual value, do not use the CLT. Use the distribution of its random variable.
- From the central limit theorem, we know that as n gets larger and larger, the sample means follow a normal distribution. The larger n gets, the smaller the standard deviation gets.

Ex. 1: A study involving stress is conducted among the students on a college campus. The stress scores follow a uniform distribution with the lowest stress score equal to one and the highest equal to five. Using a sample of 55 students, find the following.

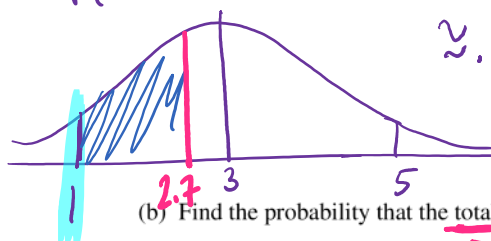
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
 $\mu_{\bar{x}} = \mu$

$n=55$ normalcdf (low, high, μ , σ)

(a) Find the probability that the mean stress score is less than 2.7: $P(\bar{x} < 2.7)$.

$P(\bar{x} < 2.7) = \text{normalcdf}(1, 2.7, 3, \frac{1.15}{\sqrt{55}})$

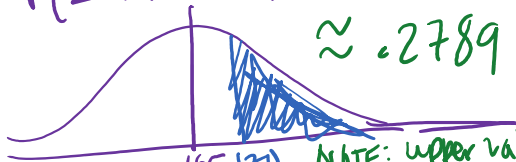
$\approx .0265$ $\bar{x} \sim N(3, \frac{1.15}{\sqrt{55}})$



2.65% chance of sample having a mean score less than 2.7

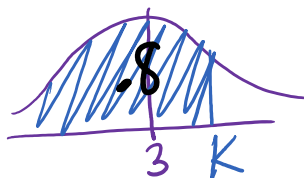
(b) Find the probability that the total stress score is greater than 170: $P(\Sigma x > 170)$.

$P(\Sigma x > 170) = \text{normalcdf}(170, 5(55), 55(3), 1.15\sqrt{55})$
 $\approx .2789$



NOTE: upper value is not 170 here. Upper value is $5(55) = 275$

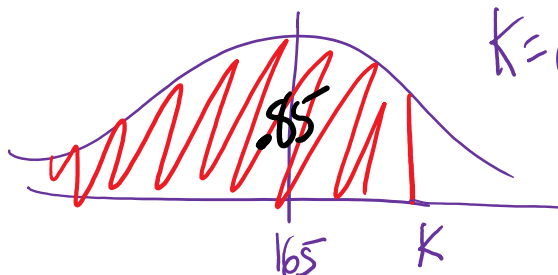
(c) Find the 80th percentile for the mean of 55 scores.



$K = \text{invNorm}(.8, 3, \frac{1.15}{\sqrt{55}})$
 $K = 3.1305$

$\bar{x} \sim N(3, \frac{1.15}{\sqrt{55}})$

(d) Find the 85th percentile for the sum of 55 scores.



$K = \text{invNorm}(.85, 55(3), 1.15\sqrt{55})$

≈ 173.84

2 decimal places

$\Sigma x \sim N(55(3), 1.15\sqrt{55})$

$X \sim U(1, 5)$ $\mu = \frac{a+b}{2}$

$\mu_x = \frac{1+5}{2} = 3$

$\sigma_x = \sqrt{\frac{(b-a)^2}{12}} = \sqrt{\frac{(5-1)^2}{12}} = \sqrt{\frac{16}{12}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} \approx 1.15$

≈ 1.15



$$X \sim \text{Exp}(\lambda) \quad \lambda = \frac{1}{\mu} \quad \mu = \sigma$$

Ex. 2: Suppose that a market research analyst for a cell phone company conducts a study of their customers who exceed the time allowance included on their basic cell phone contract; the analyst finds that for those people who exceed the time included in their basic contract, the excess time used follows an exponential distribution with a mean of 22 minutes.

$$n = 144$$

Consider a random sample of 144 customers who exceed the time allowance included in their basic cell phone contract. Let X = the excess time used by one INDIVIDUAL cell phone customer who exceeds his contracted time allowance.

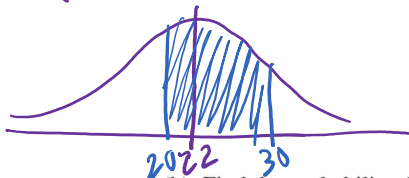
$X \sim \text{Exp}\left(\frac{1}{22}\right)$. From previous chapters, we know that $\mu = 22$ and $\sigma = 22$.

Normal

Let $\bar{X}$ = the mean excess time used by a sample of $n = 144$ customers who exceed their contracted time allowance. $\bar{X} \sim N\left(22, \frac{22}{\sqrt{144}}\right)$ by the central limit theorem for sample means.

(a) Find the probability that the mean excess time used by the 144 customers is between 20 and 30: $P(20 < \bar{x} < 30)$.

$$P(20 < \bar{X} < 30) = \text{normalcdf}(20, 30, 22, \frac{22}{\sqrt{144}}) \approx 0.8623$$



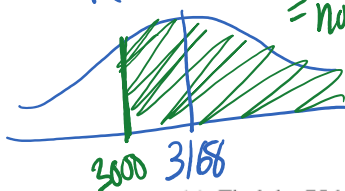
Probability that mean excess time is b/t 20 & 30 is 0.8623

(b) Find the probability that the sum of excess time used by the 144 customers is at least 3000: $P(\sum X \geq 3,000)$.

$$P(\sum X \geq 3000)$$

$$\sum X \sim N(144(22), \sqrt{144}(22))$$

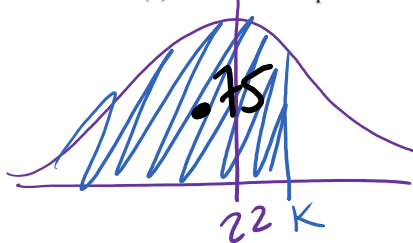
$$= \text{normalcdf}(3000, 1E99, 144(22), \sqrt{144}(22)) \approx .7377$$



(c) Find the 75th percentile for the sample mean excess time of 144 customers.

$$\bar{X} \sim N(22, \frac{22}{\sqrt{144}})$$

$$K = \text{invNorm}(.75, 22, \frac{22}{\sqrt{144}}) \approx 23.236$$



(d) Find the 85th percentile for the sum of 144 excess times used by customers.

$$\sum X \sim N(144(22), \sqrt{144}(22))$$

$$K = \text{invNorm}(.85, 144(22), \sqrt{144}(22)) \approx 3441.6184 \text{ gives the 85th percentile for the sum of excess times in the sample of 144.}$$

